

A Brief History of Downside Risk Measures

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“A man who seeks advice about his actions will not be grateful for the suggestion that he maximize his expected utility.” Roy(1952)

Introduction

There has been a controversy in this journal about using downside risk measures in portfolio analysis. The downside risk measures supposedly are a major improvement over traditional portfolio theory. That is where the battle lines clashed when Rom and Ferguson (1993, 1994b) and Kaplan and Siegel (1994a, 1994b) engaged in a “tempest in a teapot”. I should confess that I am strong supporter of downside risk measures and have used them in my teaching, research and software for the past two decades. Therefore, you should keep that bias in mind as you read this article.

One of the best means to understand a concept is to study the history of its development. Understanding the issues facing researchers during the development of a concept results in better knowledge of the concept. The purpose of this paper is to provide an understanding of the measurement of downside risk.

First, it helps to define terms. Portfolio theory is the application of decision-making tools under risk to the problem of managing risky investment portfolios. There have been numerous techniques developed over the years in order to implement the theory of portfolio selection. Among the techniques are the downside risk measures. The most commonly used downside risk measures are the *semivariance* (special case) and the *lower partial moment* (general case). The major villain in the downside risk measure debate is the variance measure as used in mean-variance optimization. It is helpful to remember that mean-variance as well as mean-semivariance optimizations are simply techniques that make up the toolbox that we call portfolio theory. In addition, the semivariance has been used in academic research in portfolio theory as long as the variance. As such, there is no basis for labeling the use of downside risk measures “post-modern portfolio theory” except for marketing “sizzle”.

The Early Years

While there was some work on investment risk earlier (Bernstein,1996), portfolio theory along with the concept of downside risk measures started with the publication of two papers in 1952. The first by Markowitz(1952) provided a quantitative framework for measuring portfolio risk and return. Markowitz developed his complex structure of equations after he was struck with a notion that “you should be interested in risk as well as return.”¹ Markowitz used mean returns, variances and covariances to derive an efficient frontier where every portfolio on the frontier maximizes the expected return for a given variance or minimizes the variance for a given expected return. This is usually called the EV criterion where E is the expected return and V is the variance of the portfolio. The important job of picking one portfolio from the efficient frontier for investment purposes was given to a rather abstract idea, the quadratic utility function.

The investor has to make a tradeoff between risk and return. The investor’s sensitivity to changing wealth and risk is known as a utility function. Unfortunately, the elements that determine a utility function for a biological system that we call a human being are obscure.

The second paper on portfolio theory published in 1952 was by Roy(1952). Roy’s purpose was to develop a practical method for determining the best risk-return tradeoff as he did not believe that a mathematical utility function could be derived for an investor. As stated above, an investor will not find it practical to maximize expected utility.²

Roy states that an investor will prefer safety of principal first and will set some minimum acceptable return that will conserve the principal. Roy called the minimum acceptable return the disaster level and the resulting technique is the Roy safety first technique. Roy stated that the investor would prefer the investment with the smallest probability of going below the disaster level or target return. By maximizing a reward to variability ratio, $(r - d)/s$, the investor will choose the portfolio with the lowest probability of going below the disaster level, d , given an expected mean return, r , and a standard deviation, s . Although Roy is not a familiar name because he finished second (his paper was published three months after Markowitz's paper), he provides a very useful tool, i.e., the reward to variability ratio computed using a disaster level return. In fact, Markowitz(1987) states that if Roy's objective had been to trace out mean-variance efficient sets using the reward to variability ratio, we would be calling it Roy's portfolio theory since Markowitz did not develop a general portfolio algorithm for selecting efficient sets until 1956 (Markowitz, 1956).

In the meantime, Roy's concept of an investor preferring safety of principal first when dealing with risk is instrumental in the development of downside risk measures. The reward to variability ratio allows the investor to minimize the probability of the portfolio falling below a disaster level, or for our purposes, a target rate of return.

Markowitz(1959) recognized the importance of this idea. He realized that investors are interested in minimizing downside risk for two reasons: (1) only downside risk or safety first is relevant to an investor and (2) security distributions may not be normally distributed. Therefore a downside risk measure would help investors make proper decisions when faced with nonnormal security return distributions. Markowitz shows that when distributions are normally distributed, both the downside risk measure and the variance provide the correct answer. However, if the distributions are not normally distributed only the downside risk measure provides the correct answer. Markowitz provides two suggestions for measuring downside risk: a semivariance computed from the mean return or below-mean semivariance (SVm) and a semivariance computed from a target return or below-target semivariance (SVt). The two measures compute a variance using only the returns below the mean return (SVm) or below a target return (SVt). Since only a subset of the return distribution is used, Markowitz called these measures partial or semi- variances.

$$SVm = \frac{1}{K} \sum_{T=1}^K \text{Max}[0, (E - R_T)]^2, \text{ below-mean semivariance} \quad (1a)$$

$$SVt = \frac{1}{K} \sum_{T=1}^K \text{Max}[0, (t - R_T)]^2, \text{ below-target semivariance} \quad (1b)$$

R_T is the asset return during time period T , K is the number of observations, t is the target rate of return and E is the expected mean return of the asset's return. The maximization function, **Max**, indicates that the formula will square the larger of the two values, 0 , or $(t - R_T)$.

After proposing the semivariance measure, Markowitz(1959) stayed with the variance measure because it was computationally simpler. The semivariance optimization models using a cosemivariance matrix (or semicovariance if that is your preference) require twice the number of data inputs than the variance model. *With the lack of cost-effective computer power and the fact that the variance model was*

already mathematically very complex, this was a significant consideration until the 1980s with the advent of the microcomputer.

Research on the semivariance did continue in the 1960s and early 1970s. Quirk and Saposnik (1962) demonstrated the theoretical superiority of the semivariance versus the variance. Mao (1970) provided a strong argument that investors will only be interested in downside risk and that the semivariance measure should be used. Unfortunately, the two semivariance models cause a lot of confusion. The main culprit is the below-mean semivariance (SV_m) which many researchers assumed is the only semivariance measure. It seems a few researchers found that the below-mean semivariance is helpful in testing for skewed probability distributions. By taking the variance and dividing it by the below-mean semivariance (SV_m), a measure of skewness resulted. If the distribution is normally distributed then the semivariance should be one-half of the variance. (In fact, some researchers call the SV_m measure a “half variance”.) If the ratio is equal to 2, then the distribution is symmetric. If the ratio is not equal to 2, then there is evidence that the distribution is skewed or asymmetric. Skewness is a measure of the symmetry of the distribution. If there is no skewness, then the distribution is symmetric. If there is significant skewness, then the distribution is asymmetric. When the skewness of an asset return distribution is negative, then the downside returns will have a larger magnitude of returns than the upside returns, i.e. losses when they occur will tend to be large losses. When the skewness of the distribution is positive, then the upside returns will have a larger magnitude of returns than the downside returns. (When losses occur, they will be smaller and when gains occur, they will be greater.)

The confusion over the semivariance still exists today with Balzer(1994) and Sortino and Price(1994) using terms such as *relative semivariance* and *downside deviation* for the *below-target return semivariance* and the term *semivariance* for the *below-mean return semivariance*. The preferred terms are usually terms that describe the measure simply and accurately; therefore, this paper utilizes the *below-mean semivariance (SV_m)* and the *below-target semivariance (SV_t)* as defined in (1a) and (1b) as the appropriate names.

Also during the 1960s, researchers moved forward using Roy’s reward to variability (R/V) ratio. It proved useful in evaluating mutual fund performance (Sharpe, 1966) on a risk-return basis.³ Because of the success of the Sharpe ratio (Professor Sharpe himself refers to this ratio as the reward to variability ratio in his research. He acknowledges that it is commonly known as the “Sharpe ratio” but feels that it is not the appropriate name since he did not develop the ratio.⁴), Treynor and Jensen developed their risk-return performance measures as derivations from the Capital Asset Pricing Model (CAPM).

The Modern Era of Semivariance Research

Because these performance measures depend on a normal distribution, researchers started to question them in the 1970s. Studies by Klemkosky(1973) and by Ang and Chua(1979) showed that these measures could provide incorrect rankings and suggested the reward to semivariability (R/SV) ratio as an alternative. (Note that the R/SV ratio is really the return to below-target semideviation ratio. The semideviation simply being the square root of the semivariance.) By taking the excess return (r-d) and dividing by the standard deviation, the R/V ratio is standardized. Therefore, there should be no statistical relationship between the ratio (R/V) and the risk measure, standard deviation. Both studies performed cross-sectional regression studies between the performance measure and the risk measure for a large sample of mutual funds. If the r-square of the regression is close to zero, then the ratio is statistically independent of the risk measure and, therefore, statistically unbiased. A summary of these two studies is in Table 1. Note that in both studies, the traditional measures (R/V ratio, Treynor and Jensen measures) are statistically related (high r-squares) to their underlying risk measure, the standard deviation or the beta. However, the relationship between the reward to semivariability ratio and the semideviation has the lowest r-square in either study.

Table 1 - Summary of Klemkosky (1973) and the Ang and Chua (1979) Studies. The Results are the R-Squares of the Regression between the Return-Risk Ratio and Its Risk Measure. Low R-Squares indicate that the risk-return performance measure is statistically unbiased.

	Klemkosky	Ang and Chua	
Number of Mutual Funds	40	111	
Time Period of Sample	1966-1971	1955-1974	
Number of Observations	24	77	
Length of Holding Period	1	1	4
<hr/>			
Returns in Quarters			
<hr/>			
R/V vs. Standard Deviation	.16 (2.72)	.60 (8.60)	.60 (8.50)
Treynor R/B vs. Beta	.27 (3.75)	.67 (7.30)	.65 (7.80)
Jensen Alpha vs. Beta	.41 (5.13)	.95 (2.40)	.94 (2.70)
R/SVm vs. Half Variance (SVm)	.04 (1.23)	.38 (8.25)	.39 (8.30)
R/SVt vs. Semideviation (SVt)		.23 (5.70)	.04 (2.20)
R/MAD vs. Mean Absolute Deviation (MAD)	.12 (2.28)		
<hr/>			
(T-tests of Slope Coefficient in Parenthesis)			
Source: Klemkosky (1973) and Ang and Chua (1979)			
<hr/>			

These studies provide the strongest support for the below-target semivariance measure (SVt). The Ang and Chua(1979) results are stronger because of the larger data and mutual fund samples. However, the same pattern of results is evident in both studies. Ang and Chua (1979, footnote 2) also present a formal utility theory proof that the below-target semivariance (SVt) is superior to the mean semivariance (SVm). Note that Klemkosky(1973) did not test the target semivariance (SVt) and the Ang and Chua(1979) did not test the mean absolute deviation (MAD). If the security distributions are normally distributed both the R/SV and R/V ratios will provide the best answer. However, if the distributions are not normally distributed, then R/V ratio is statistically biased while the R/SV ratio will still be providing the correct answer.

Study of the below-target semivariance measure continued with Hogan and Warren(1972). Hogan and Warren provide an optimization algorithm for developing expected return (E) – below-target semivariance (S) efficient portfolios, thus the ES criterion. Hogan and Warren(1974) also provide an interesting diversion. They developed a below-target semivariance capital asset pricing model (ES-CAPM). With the CAPM, there is no interest in the below-mean semivariance (SVm) since asset distributions are assumed to be normal (symmetric). In this case, the SVm measure is simply the “half variance”. The SVt version of the ES-CAPM is of interest if the distributions are nonnormal and non-symmetric (asymmetric). Nantell and Price(1979) and Harlow and Rao(1989) further extended the SVt version of the ES-CAPM into the more general lower partial moment (LPM) version, EL-CAPM.

However, in the early 1970s, the mean semivariance (SVm) was still used by many researchers as evidenced by the Klemkosky (1973) article. It was at this point that Burr Porter became interested in the semivariance. He had helped develop computer algorithms for doing stochastic dominance analysis (Porter, Wart and Ferguson, 1973). Stochastic dominance is a very powerful risk analysis tool.⁵ It converts the probability distribution of an investment into a cumulative probability curve. Next, math analysis of the cumulative probability curve is used to determine if one investment is superior to another investment. Stochastic dominance has two major advantages: It works for all probability distributions and it includes all possible risk averse utility assumptions. Its major disadvantage? An optimization algorithm for selecting stochastic dominance efficient portfolios has never been developed. (Although later, Bey (1979) proposes a E-SVt algorithm to *approximate* the second degree stochastic dominance efficient portfolio sets.)

A mean-semivariance computer program developed in 1971 by a research team directed by Professor George Philippatos at Penn State University piqued Porter’s (1974) interest in the semivariance.⁶ The Penn State E-S program (as well as a mean-variance program) was developed using information from Markowitz’s (1959) book. This E-S program used the below-target semivariance (SVt) and reward to semivariability (R/SVt) ratios. Believing that the below-mean semivariance (SVm) to be the appropriate measure, Porter tested the two measures using his stochastic dominance program. Surprisingly, the tests showed that below-target semivariance (SVt) portfolios were members of stochastic dominance efficient sets (The SVt portfolios were superior investments), while the below-mean semivariance (SVm) portfolios were not. Porter also demonstrated that mean variance (EV) portfolios were included in the stochastic dominance efficient sets. Porter and Bey(1974) followed with a comparison of mean-variance and mean-semivariance optimization algorithms.

The Birth of the Lower Partial Moment (LPM)

Every once in a great while, there is a defining development in research that clarifies all issues and gives the researcher an all-encompassing view. This development in the research on downside risk measures occurred with the development of the Lower Partial Moment (LPM) risk measure by Bawa (1975) and Fishburn (1977). Moving from the semivariance to the LPM is equivalent to progressing from a silent

black and white film to a wide screen Technicolor film with digital surround sound. This measure liberates the investor from a constraint of having only one utility function, which is fine if investor utility is best represented by a quadratic equation (variance or semivariance). Lower Partial Moment represents a significant number of the known Von Neumann-Morgenstern utility functions. Furthermore, the LPM represents the whole gamut of human behavior from risk seeking to risk neutral to risk aversion. The LPM is analogous to Mandelbrot's development of fractal geometry. The fractal geometry eliminates the limitation of traditional geometry to 0, 1, 2 and 3 dimensions. With fractal geometry, the 2.35 dimension or the 1.72 dimension is open to practical exploration. (It should be noted that cynics looking at Mandelbrot sets might wonder what advantage there is to having the mathematical formula for Paisley patterns. For one, computerized looms will have no trouble manufacturing the material for Paisley ties.) There is no limit to the dimensions that can be explored with fractal geometry. As in fractal geometry, the LPM eliminates the semivariance limitation of a single risk aversion coefficient of 2.0. Whether we wish to explore a risk aversion coefficient of 1.68 or a risk aversion coefficient of 2.79, or a risk-loving coefficient of 0.81, there is no limitation with the LPM.

Coincidentally, Burr Porter was auditing a course on utility theory from Peter Fishburn. Because of Porter's work on the below-target semivariance, Fishburn became interested in the measure. While Fishburn was developing his thoughts on the subject, Vijay Bawa (1975) published his seminal work on lower partial moment that defined the relationship between lower partial moment and stochastic dominance. Bawa (1975) was the first to define lower partial moment (LPM) as a general family of below-target risk measures, one of which is the below-target semivariance. The LPM describes below-target risk in terms of risk tolerance. Given an investor risk tolerance value \mathbf{a} , the general measure, the lower partial moment, is defined as:

$$\text{LPM}(\mathbf{a}, \mathbf{t}) = \frac{1}{\mathbf{K}} \sum_{\mathbf{T}=1}^{\mathbf{K}} \text{Max}[0, (\mathbf{t} - \mathbf{R}_{\mathbf{T}})]^{\mathbf{a}}, \quad (2)$$

where \mathbf{K} is the number of observations, \mathbf{t} is the target return, \mathbf{a} is the degree of the lower partial moment, $\mathbf{R}_{\mathbf{T}}$ is the return for the asset during time period \mathbf{T} , and Max is a maximization function which chooses the larger of two numbers, 0 or $(\mathbf{t} - \mathbf{R}_{\mathbf{T}})$. It is the \mathbf{a} value that differentiates the LPM from the SVt. Instead of squaring deviations and taking square roots as we do with the semivariance calculations, the deviations can be raised to the \mathbf{a} power and the \mathbf{a} root can be computed. There is no limitation to what value of \mathbf{a} that can be used in the LPM except that we have to be able to make a final calculation, i.e., the only limitation is our computational machinery. The \mathbf{a} value does not have to be a whole number. It can be fractional or mixed. It is the myriad values of \mathbf{a} that make the LPM wide screen Technicolor to the semivariance's black and white. Consider that utility theory is not used solely to select a portfolio from an efficient frontier. It is also used to describe what an investor considers to be risky. There is a utility function inherent in every statistical measure of risk. We can't measure risk without assuming a utility function. The variance and semivariance only provide us with one utility function. The LPM provides us with a whole rainbow of utility functions. This is the source of the superiority of the LPM risk measure over the variance and semivariance measures.

Bawa(1975) provides a proof that the LPM measure is mathematically related to stochastic dominance for risk tolerance values (\mathbf{a}) of 0, 1, and 2. The LPM $\mathbf{a}=0$ is sometimes called the below target probability (BTP). We will see later in Fishburn's (1977) work that this risk measure is appropriate only for a risk loving investor. LPM $\mathbf{a}=1$ has the unmanageable name of the average downside magnitude of failure to meet the target return (expected loss). Again, the name of this measure is misleading because LPM $\mathbf{a}=1$ assumes an investor who is neutral towards risk and, in actuality, is a very aggressive investor. LPM $\mathbf{a}=2$ is the semivariance measure, which is sometimes called the below target risk (BTR) measure. This name

is more appropriate to portfolio selection than the other measures, since it actually measures below target risk and is consistent with a risk averse investor.

Table 2 provides an EXCEL spreadsheet for computing the Lower Partial Moment for degree 2.0 and a target return of 15%. The EXCEL spreadsheet can also be used to calculate the LPM values in Table 3. The interested reader may find Tables 2 and 3 an excellent learning tool for understanding the LPM calculation. The stock in Table 2 has two states of nature. In the first state of nature, the stock has an 80% probability of earning a 10% rate of return and during the second state, it has a 20% probability of earning a 35% rate of return. The mean of the distribution is 15% and in this case the target return (t) is the same as the mean. However, this is only for this demonstration example. In actual use, the target return will normally be different from the mean return. The formula starting in the E6 cell is in the table in order to show the formulae used from E7 to E16. The actual formula would be entered into the E7 cell and copied to cells E8 to E16. Similarly, the formula listed in the E19 cell is used in the E18 cell to compute the final value of the lower partial moment.

Table 2 - EXCEL Worksheet for Computing LPM Degree 2 and a target value of 15%

A	1 B	C	D	E	F	G	H
	2			Lower Partial Moment Calculation (a,t)			
	3	Target Rate of Return		15			
	4	Degree of LPM:		2			
	5						
	6	Period	Return	IF(\$D\$3-C7>0.(\$D\$3-C7)^\$D\$4.0)			
	7	1	10		25		
	8	2	10		25		
	9	3	10		25		
	10	4	10		25		
	11	5	10		25		
	12	6	10		25		
	13	7	10		25		
	14	8	10		25		
	15	9	35		0		
	16	10	35		0		
	17						
	18	Lower Partial Moment			20		
	19			SUM(E7:E16)*(1/COUNT(E7:E16))			
	20						

Cells B7 to B16 denote 10 periods representing the probabilities of the returns for the security. Cells C7 to C16 contain the returns for each period. Cell E6 contains the formula that is used in Cells E7 to E16. The formula in Cell E19 is placed in E18 and is used to calculate the final value of the LPM, which is 20. (Cells E6 and E19 would be left blank in the spreadsheet program.)

Source: Silver (1993)

Fishburn (1977) extends the general LPM model to the (\mathbf{a}, \mathbf{t}) model, where \mathbf{a} is the level of investor risk tolerance and \mathbf{t} is the target return. Fishburn provides the unlimited view of LPM with fractional degrees of 2.33 or 3.89. Given a value of the target return, \mathbf{t} , Fishburn demonstrates the equivalence of the LPM measure to stochastic dominance for all values of $\mathbf{a} > 0$. Fishburn also shows that the \mathbf{a} value includes all types of investor behavior. The LPM value $\mathbf{a} < 1$ captures risk seeking behavior. Risk neutral behavior is $\mathbf{a} = 1$, while risk averse behavior is $\mathbf{a} > 1$. The higher the \mathbf{a} value is above a value of one, the higher the risk aversion of the investor. Table 3 demonstrates the behavior of the LPM measure for different degrees (\mathbf{a}). The target return is set to 15% which for this example is the same as the mean return for the two investments. Normally, the target return will not be equal to the mean return. In the ES-CAPM and EL-CAPM models of Hogan and Warren (1974) and Nantell and Price (1979), the target return is set to the risk free Treasury bill rate. (Remember that the EXCEL spreadsheet in Table 2 can be used to compute the LPM values contained in Table 3.)

Note that when $\mathbf{a} < 1$, the Investment A is considered to be less risky than Investment B, although the skewness number and the distribution information indicates that Investment B has less downside risk. Note that this is consistent with a risk loving utility function. When $\mathbf{a} > 1$, then Investment B is less risky than Investment A. Note that this is consistent with a risk averse utility function. Also, as \mathbf{a} increases, Investment A takes on a heavier risk penalty. When $\mathbf{a} = 1.5$, Investment A is twice as risky as Investment B. When $\mathbf{a} = 2.0$, Investment A is four times as risky as Investment B. When $\mathbf{a} = 3$, Investment A is sixteen times as risky as Investment B.⁷ *This demonstrates the importance of setting the correct value of \mathbf{a} when using the LPM downside risk measures.*

Table 3 - Example of Degrees of the Lower Partial Moment

	Company A		Company B	
	Return	Prob.	Return	Prob.
	-5.00	0.20	10.00	0.80
	20.00	0.80	35.00	0.20
Mean Return	15.00		15.00	
Variance	100.00		100.00	
Skewness	-1.50		1.50	
LPM a=0.0 t=15	0.20		0.80	
LPM a=0.5 t=15	0.89		1.79	
LPM a=1.0 t=15	4.00		4.00	
LPM a=1.5 t=15	17.89		8.94	
LPM a=2.0 t=15	80.00		20.00	
LPM a=3.0 t=15	1600.00		100.00	

Source: Silver(1993)

Note: When $a=1.0$ and t is equal to the mean return, then the two LPM values are equal. If t is set to some other return, then the LPM values will depend on the degree of skewness in the return distribution.

Utility Theory or the Maximization of Economic Happiness

When academics discuss utility theory (theory of economic satisfaction), they usually are referring to the von Neumann and Morgenstern(1944) utility functions. The Fishburn family of LPM utility functions assert that the investor is risk averse (or risk seeking depending on the value of \mathbf{a}) below the target return and risk neutral above the target return. This utility function is a combination of being very conservative with the downside risk potential of a portfolio and very aggressive with the upside potential of a portfolio. Fishburn(1977, pp.121-2) examines a large number of von Neuman and Morgenstern utility functions that have been reported in the investment literature and finds a wide range of \mathbf{a} values ranging from less than 1 to a value greater than 4. The $\mathbf{a}=2$ target semivariance utility function was not commonly found. Given this result, Fishburn concludes that the generalized $\mathbf{a-t}$ (LPM) model is superior to the target semivariance because it is more flexible at matching investor utility.

There are two important caveats discussed by Fishburn. The first caveat forms the basis of the Kaplan and Siegel(1994a) argument against using the target semivariance measure. They argue that the investor utility function measured by the target semivariance is linear above the target return. Since this implies that investors are risk neutral above the target return, the risk measure is of limited importance.⁸ Fishburn (1977) searched the usage of utility functions in the investment literature and found that approximately one third of the utility functions are linear above the target return. The rest of the utility functions differed only above the target return. This is problematic only if the investor is concerned with above-target returns. However, researchers from Roy(1952) to Markowitz(1959) to Swalm(1966) to Mao(1970) argue that investors are not concerned with above-target returns and that the semivariance is more consistent with risk as viewed by financial and investment managers.. In addition, when one third of all utility functions known within the utility theory literature are Fishburn LPM utility functions, this represents a considerable number. The variance and its utility function represent only one utility function.

Fishburn's second caveat is seldom mentioned within the downside risk literature although economists commonly call it the decreasing marginal utility of wealth. Very simply, an additional dollar of income to a wealthy person provides less economic happiness (utility) than an additional dollar of income to a poor person. Concerning the LPM ($\mathbf{a,t}$) measure, the risk aversion coefficient, \mathbf{a} , is dependent on the amount of the investor's total wealth. If the amount of wealth at risk is very small relative to the investor's total wealth then the investor can be very aggressive in terms of investing in risky investments (low values of \mathbf{a}). If the amount of money at risk is a substantial portion of the investor's total wealth, then the investor will be more risk averse (higher values of \mathbf{a}).

Laughunn, Payne and Crum (1980) developed an interactive computer program in BASIC that utilizes Fishburn's (1977) methodology for estimating the value of \mathbf{a} for an individual. They study 224 corporate middle managers by giving them a number of small investment projects from which to choose. They find that 71% of the managers exhibit risk-seeking behavior ($\mathbf{a}<1$). Only 9.4% of the managers have \mathbf{a} values around 2. Only 29% of the managers were risk averse ($\mathbf{a}>1$).

Next, they study the impact of ruinous loss. Without the threat of ruinous loss, most managers are risk seeking. However, if the investment projects include ruinous losses, there is a statistically significant shift to risk averse behavior by the managers. With the chance of a ruinous loss, the majority of the corporate managers were risk averse. ***Therefore, the estimation of the investors risk coefficient, \mathbf{a} , is dependent on the relationship between the value of the investment portfolio and the investor's total wealth.***

In order to provide investment advice, the use of an appropriate risk measure is imperative. The factors affecting the choice of the risk measure are:

- *Investors perceive risk in terms of below target returns.*
- *Investor risk aversion increases with the magnitude of the probability of ruinous losses.*
- *Investors are not static. As the investor's expectations, total wealth, and investment horizon changes, the investor's below target return risk aversion changes. Investors have to be constantly monitored for changes in their level of risk aversion.*

Using LPM Measures Means Algorithms (Algorithm Research)

Algorithms are cookbook recipes that are programmed into computers to solve complex mathematical problems. In portfolio theory, the complex problem is deciding which investments receive which proportion of the portfolio's funds. There are two types of algorithms: optimal and heuristic. Optimal algorithms provide the **BEST** answer given the input information. Heuristic algorithms provide a **GOOD** (approximately correct) answer given the input information. Heuristic algorithms are attractive because they provide answers using fewer computational resources, that is, they provide answers cheaper and faster than an optimal algorithm.

Both optimal and heuristic algorithms have been shown to work with the LPM(**a,t**) model. The original Philippatos (1971)-Hogan and Warren (1972) E-SVt optimization algorithm was extensively tested by Porter and Bey (1974), Bey (1979), Harlow (1991), and Nawrocki (1991,1992). The major issue concerning LPM algorithms is their ability to manage the skewness of a portfolio. There are two concerns: managing skewness during a past historic period and managing skewness during a future holding period. These separate concerns arise as academics are concerned with explaining how things work by studying the past while practitioners are interested in how things are going to work in the future.

Table 4 presents the results of optimizing the monthly returns of 20 stocks utilizing the Markowitz E-V algorithm and the Philippatos-Hogan and Warren E-SVt algorithm. The following characteristics of LPM portfolios should be noted as the degree of the LPM is increased from **a=1** (risk neutral) to **a=4** (very risk averse). These results are historic (looking backward) results.

- Each portfolio selected has an approximate expected monthly return of 2.5%. The Markowitz (1959) critical line algorithm does not provide portfolios for specific returns. Effectively, it samples the efficient frontier by generating corner portfolios where a security either enters or leaves the portfolio.
- The LPM portfolios have higher standard deviations than the EV portfolio. This result should not be a surprise as the EV algorithm optimizes the standard deviation.
- The risk neutral (**a=1.0**) LPM portfolio has a higher semideviation (**a=2.0**) than the EV portfolio and the risk averse (**a > 1.0**) LPM portfolios.
- The risk averse (**a > 1.0**) LPM portfolios have lower semideviations than the EV portfolio. This result should not surprise as the E-SVt algorithm optimizes the LPM measure.
- Each of the LPM portfolios has increased skewness value compared to the EV portfolio. As the degree of the LPM increases, the skewness increases. The skewness values are statistically significant.
- The LPM optimizer is capturing co-skewness effects as the skewness of the risk averse (**a > 1.0**) LPM portfolios is higher than the skewness values of any individual stock.
- The R/V ratios are lower for the LPM portfolios than for the EV portfolio. Again, no surprise here.
- The R/SV ratios are higher for the risk averse (**a > 1**) LPM portfolios than for the EV portfolio. Again, no surprise.
- The LPM portfolios are different from the EV portfolios in the security allocations ranging from 17% to 30% difference. The higher the LPM degree the greater the difference in security allocations between the LPM portfolio and the EV portfolio. As the LPM portfolio is trying to optimize the portfolio using an increasingly risk averse utility function, the increase in the difference between the allocations is expected.

- Skewness can be diversified away. In order to maintain skewness in a portfolio, the LPM portfolios will usually contain fewer stocks than a comparable EV portfolio. Note as the degree of the LPM increases the skewness increases, and the number of stocks in the portfolio decreases.
- Note how the allocation in Consolidated Edison (The stock with the highest skewness value) increases as the degree of the LPM increases.
- These results are also obtained from larger security samples and time period samples (Nawrocki, 1992).
- The portfolios are selected from their respective efficient frontiers (**a = 1.0 to 4.0**). Each frontier is a subset of the mean-target semivariance (ES) feasible region but only the LPM (**a = 2.0**) portfolio will be on the efficient frontier for this feasible region.

Table 4 - An In-Sample Comparison of An EV Optimal Portfolio With Comparable LPM Optimal Portfolios Using Markowitz (1959) Critical Line Algorithm with 48 Monthly Returns (1984-1987) for Twenty Stocks.

A L L O C A T I O N S						
Security	Security Skewness	LPM 1.0	LPM 2.0	LPM 3.0	LPM 4.0	EV
Adams Millis	0.4336	5.3019	6.6483	7.3437	8.0512	9.1183
Allegheny Pwr.	0.5610					.6342
Belding Hemin.	0.4290		2.0557	3.1404	2.6374	
Con. Edison	0.7050*	50.4361	59.5337	59.5219	60.8875	35.0428
Con. Nat. Gas	0.3520	1.5136	1.7933	1.4464		1.9510
FMC Corp.	0.5406	2.2994	5.5741	6.9885	7.3007	5.6800
Heinz, H.J.	0.3892	9.4730	12.1067	14.2124	16.9302	17.1882
Idaho Power	0.2327	5.2687				3.4108
Kansas Power	0.4962	12.8200	8.6804	7.3462	4.1953	12.9142
Mercantile St	-0.4134	12.8788	3.6086			13.0554
Total Allocations		100.0000	100.0000	100.0000	100.0000	100.0000
# Securities		8	8	7	6	9

Portfolio Statistics						
Portfolio		LPM 1.0	LPM 2.0	LPM 3.0	LPM 4.0	EV
Return		2.5140	2.5001	2.4849	2.5197	2.5062
Std.Deviation		3.5233	3.5443	3.5290	3.6703	3.3838
SemiDeviation		1.0949	0.9705	0.9514	0.9953	1.0143
Skewness		.6381*	.7861*	.7863*	.7912*	.5456
Beta		.2976	.2888	.2952	.3012	.3848
R/V Ratio		.5754	.5680	.5662	.5539	.5968
R/SV Ratio		1.8516	2.0746	2.1102	2.0426	1.9910
% Difference LPM Vs. EV		17.2589	26.5887	28.9257	30.0991	

* - Significant skewness at 2 standard deviations (s = .3162)

Source: Nawrocki (1992).

The LPM heuristic algorithm came along later. Nawrocki(1983) developed a linear programming (LP) LPM heuristic algorithm utilizing reward to lower partial moment (R/LPM $\mathbf{a,t}$) ratios. This heuristic algorithm derives from earlier work on portfolio theory heuristic algorithms by Sharpe(1967) and Elton, Gruber and Padberg(1975). Similar to the Sharpe heuristic, this heuristic assumes that the average correlation between securities is zero. As a result, this algorithm requires a large number of stocks in order to obtain a degree of diversification comparable to optimal algorithms. Besides the lower computational costs, heuristic algorithms can provide better forecasting results (Elton, Gruber and Urich, 1978).

Nawrocki and Staples (1989) and Nawrocki (1990) provide extensive *out-of-sample* backtests of the R/LPM heuristic algorithms. One of the important findings is that the skewness of the portfolio can be managed using the R/LPM algorithm. As demonstrated in Table 3, a risk averse investor prefers positive skewness to negative skewness. Over a 30 year testing period (1958-1987), Nawrocki (1990) finds a direct relationship between the \mathbf{a} parameter in the LPM($\mathbf{a,t}$) measure and the skewness of the portfolio (As \mathbf{a} increases, the skewness increases). An algorithm employing the LPM($\mathbf{a,t}$) is an alternative to purchasing puts, synthetic puts, or other portfolio insurance strategies. However, like any insurance policy there is a cost. The portfolio manager can increase the skewness in the portfolio but at the cost of reduced returns. It should be noted that any portfolio insurance scheme would increase the skewness of the portfolio but at the cost of reduced returns.

The Nawrocki (1990) study is presented in Table 5. These results are holding period (looking forward or forecasting) results. A random sample of 150 stocks was tested for the 30 year period using 48 month estimation periods and 24 month revision periods, i.e. every 24 months the historic period is updated to the most recent 48 months and a new portfolio is chosen. One-percent transaction costs were computed with each portfolio revision. As the LPM degree \mathbf{a} increases from 1.0 (risk neutral) and becomes risk averse, the skewness of the portfolio increases. This is the type of forecast results that makes the LPM risk measure useful to a practitioner. At degrees of \mathbf{a} above four, the skewness values are statistically significant. Typically, the higher the skewness value, the lower the downside risk of the portfolio.

However, the insurance premium concept comes into play. For degrees of \mathbf{a} up to five, the R/SV ratio remains above 0.20. The skewness values are statistically significant at this level. Unfortunately, as the skewness value increases from a value of 0.34 ($\mathbf{a} = 5.0$) to a value of 0.61 ($\mathbf{a} = 10.0$), the R/SV ratio decreases from 0.20 to 0.18 indicating reduced risk-return performance as the skewness increases.

Table 5 - Out-of-Sample Skewness Results Using R/LPM Heuristic Algorithm from 1958 to 1987 (30 Years of Monthly Data) with Portfolios Revised Every Two Years Using 48 Month Historic Periods with a Sample of 150 Stocks. The skewness results are the average of portfolios with 5, 10, and 15 stocks. The results are compared to an Optimal Mean-Variance (EV) Portfolio Strategy.

LPM Degree	Skewness	R/SVt Ratio
a		
0.0	.1122	.1712
1.0	.0756	.1984
1.2	.0719	.2117
1.4	.0713	.2221
1.6	.0833	.2110
2.0	.1110	.2089
2.8	.1934	.2186
3.0	.2115	.2155
4.0	.2771*	.2098
4.6	.3093*	.2044
5.0	.3446*	.2005
6.0	.4287*	.1905
7.0	.4975*	.1848
8.0	.5413*	.1854
9.0	.5855*	.1825
10.0	.6129*	.1794
EV Optimal	-.0546	.1383

* - Indicates statistical significance at two standard deviations

Source: Nawrocki(1990)

Contrary to Kaplan and Siegel (1994a), an investor can use the Fishburn (\mathbf{a}, \mathbf{t}) to approach a risk neutral position over time by setting the parameter \mathbf{a} to a value of 1.0 which is risk neutral. While Fishburn's utility functions were constant, he did intend the (\mathbf{a}, \mathbf{t}) model to be general enough to handle any investor in any given risk-return scenario. If investors follow life cycle investment strategies, (product, firm or individual), then the parameter \mathbf{a} can be varied in order to account for an investor who is a different individual under one risk-return scenario than under another. In addition, an investor can use a LPM heuristic or optimization algorithm to build increased skewness into a portfolio without using a put position. In other words, the LPM algorithms can be used to directly manage the skewness of the portfolio. The results do concur with Kaplan and Siegel's recommendation that the emphasis should be on changes in the mix of the assets. In the example in Table 5, the portfolios were re-computed (re-mixed) every two years. The results are *out-of-sample*. Therefore, the LPM algorithms forecast well enough to manage the future skewness of the portfolio.

Levy and Markowitz (1979) and Kroll, Levy and Markowitz (1984) make the only strong argument for the use of the quadratic utility function and mean-variance analysis. Essentially, their argument is that while the individual stock distributions are nonnormal, the optimal diversified portfolios will be very close to a normal distribution. (We know that skewness can be diversified away by as few as 6 securities in a portfolio.) Since the optimized portfolios can be approximated by a normal distribution, then the quadratic utility function can be used to maximize investor utility. As seen in Tables 4 and 5, the LPM measure can be used to build skewness into a diversified portfolio. Therefore, the LPM is necessary both at the optimization level and at the utility level because both the individual securities and the diversified portfolios will be nonnormal. In addition, Bookstaber and Clarke(1985) demonstrate that using variance to evaluate put and call (option) positions within a portfolio will be misleading. The measurement of the riskiness of an optioned portfolio has to be able to handle skewed distributions. The option positions add skewness into a portfolio, therefore, the analysis of the risk-return of such a portfolio has to be able to handle nonnormal distributions. Therefore, the LPM measure is an important tool for deciding the amount of options to add to a portfolio. (Using options and other methods to increase the skewness of a portfolio is known as portfolio insurance.)

Recent Research in the Practitioner Literature

Around 1990, the downside risk measures started to appear in the practitioner literature. Brian Rom and Frank Sortino have been the strongest supporters of the downside risk measure in the practitioner literature and have implemented it in Brian Rom's optimization software. For the most part, both are interested in the below-target semideviation and the reward-to-semivariability ratio (R/SVt). Sortino and van der Meer (1991) describe the downside deviation (below-target semideviation) and the reward-to-semivariability ratio (R/SVt) as tools for capturing the essence of downside risk. Sortino continued to contribute in areas of performance measurement (Sortino and Price, 1994) and in the estimation of the target semivariance (Sortino and Forsey, 1996). Rom has focused on educating practitioners about downside risk measures. Rom and Ferguson (1993) started the controversy on downside risk measures with an article in this journal. They followed with a spirited defense of the downside risk measure (Rom and Ferguson, 1994b) and an article summarizing the use of downside risk measures for performance measurement (Rom and Ferguson, 1997-98). Balzer (1994) and Merriken (1994) also provide very good

discussions on skewness in asset returns and the appropriateness of the semivariance and its applications. Merriken (1994) follows the lead of Bookstaber and Clarke (1985) and demonstrates how the semivariance can be used to evaluate the downside risk of different hedging strategies using stock options and interest rate swaps.

The one criticism I have of this recent work is that it has lost sight of the benefits of the lower partial moment. Almost all of these studies are concerned with the semivariance ($\mathbf{a} = 2$) and state the investor's utility solely in terms of the target return (\mathbf{t} or MAR). Unfortunately, these studies ignore the different levels of the investor's risk aversion (\mathbf{a}) that are available to the user of the lower partial moment ($\text{LPM}_{\mathbf{a},\mathbf{t}}$) and the reward-to-lower partial moment ratio ($\text{R/LPM}_{\mathbf{a},\mathbf{t}}$). The exception is Balzer (1994) who notes that higher degrees of the LPM represent higher levels of risk aversion. Sortino and Price (1994) attempt to handle different investor risk aversion coefficients by using the Fouse index, which incorporates a risk aversion coefficient into the target semivariance framework. The Fouse index is derived from Fishburn's semivariance utility function. It is simply the semivariance version of the Sharpe utility measure (Sharpe and Alexander, 1990, 717-720) which integrates a workable risk aversion coefficient into the variance framework. Functionally, the Fouse index simply replaces the Roy safety first return, \mathbf{t} , with the risk aversion coefficient. Both can be used to select different portfolios from a given frontier. Neither adjusts the risk measure or the efficient frontier to reflect the risk attitude of the investor. The basic difference is that the target, \mathbf{t} , allows us to move around the particular efficient frontier neighborhood. The risk aversion coefficient, \mathbf{a} , allows us to move around the world of efficient frontiers. It is the degree, \mathbf{a} , of the LPM that provides the full usefulness of the measure.

The other issue with the recent work is the emphasis on general asset classes. The academic research in downside risk measures has centered on the optimization of individual stocks and the performance evaluation of various portfolios, specifically mutual funds. Markowitz's (1959) optimization model was developed for selecting stock portfolios. General asset class allocation did not become an application of portfolio theory until the Wells Fargo Bank introduced asset classes in the late 1970s because of poor portfolio performance during the early 1970s. Asset allocation became the major application of portfolio theory simply because the portfolio optimization models were very complex and required very expensive computers to solve the optimization problem. Individual security allocation optimization was too expensive to be cost effective. As this cost issue coincided with the development of indexing strategies, it is easy to see why portfolio theory became asset class optimization in practitioner applications. Because the intercorrelations between individual securities are much lower than the intercorrelations between general asset classes and because portfolio skewness is diversified away in a large portfolio, the LPM models are probably much more useful with the portfolio selection of individual securities than with the broad asset classes. With the modern microcomputer, the issue of computational cost and complexity is moot. However, this is an issue that will not be settled in this paper. All one has to do is look at the hornets' nest that has been stirred up by William Jahnke's (1997) criticism of the Brinson, Hood, and Beebower (1986) article.

Resolving the Tempest in the Teapot: Academics vs. Practitioners

Have you ever heard two people arguing without listening to each other's argument? This is the case with Rom and Ferguson (1993, 1994a) and Kaplan and Siegel (1994a,b). While Rom and Ferguson (1994b) provide a very detailed and generally correct response to Kaplan and Siegel (1994a), neither side really understands the purpose or motivation of the other. There is a fundamental difference between practitioners and academics. Practitioners look to the future. They want to know what to do now that will provide an acceptable result in the future. In other words, practitioners forecast. Academics wish to understand how and why things work. The only way to explain how things work is to look backwards. An academic model of how a system works may be a very good explanatory model but a very poor forecasting model. Kaplan and Siegel's (1994b) arguments are highly restrictive academic arguments that really do not pertain to practitioners. Their arguments are appropriate only if you wish to look backwards.

Their arguments do not apply to the future because there is no empirical support for their approach as a forecasting technique. Meanwhile, Rom and Ferguson are practitioners and are looking forward.

The issue is which method will provide the best result in the future? Both parties to the argument reference one Fishburn utility function that uses the semivariance. Kaplan and Siegel's (1994) utility argument is too restrictive in that it does not allow for the large number of utility functions that are available to the investor using the LPM(\mathbf{a}, \mathbf{t}) model. Remember that Fishburn (1977) searched the literature for von Neumann and Morgenstern utility functions and found that over 30% were consistent with the LPM utility functions. The rest were consistent with Fishburn's utility measures on the downside but not on the upside. It's ironic that Rom and Ferguson's (1994) reply to Kaplan and Siegel (1994a) effectively relies on the wealth of utility functions available with the Bawa-Fishburn LPM model although in their work they use only one. In their reply, they note the range of investor behavior that can be matched with the LPM measure. LPM analysis frees the investor from a single utility function by allowing the choice of a large number of utility functions that can handle human behavior ranging from risk loving to risk neutral to risk averse behavior.

Kaplan and Siegel (1994a) also suggest that the semivariance will suffer from a small sample problem. They demonstrate this with an example using annual returns taken from the Japanese stock market. My feeling about Kaplan and Siegel's Japanese market example is that this is one of those situations if you can't say anything nice, don't say anything. Suffice to say, Rom and Ferguson (1994b, 1997-98) and Sortino and Forsey (1996) have provided solutions to the small sample problem.

In response to the Rom and Ferguson (1994) reply, Kaplan and Siegel (1994b) show their true colors and finally resort to a Capital Asset Pricing Model (CAPM) academic argument. They make the error of equating CAPM with portfolio theory in the title of their reply. Portfolio theory is a very general body of knowledge. CAPM is a highly restrictive case within portfolio theory; however, it is not portfolio theory. The problem with the academic asset pricing theory is it takes the individual out of the decision process.⁹ Buy the market index and insure it with put options. As the individual's wealth changes, change the put position. Where is the individual's utility function? It has been assumed away in the asset-pricing model. There are no utility functions in the asset pricing world, Fishburn, von Neumann and Morgenstern, or otherwise. In the real world, investors have portfolio insurance premiums, different time horizons, different wealth levels, different goals, different tax situations, and most importantly, multiple goals. The "one size fits all" approach from the academic world is not any more useful to a practitioner than the academic approach that attempts to determine one utility function for a person to maximize.¹⁰

Bawa and Fishburn's lower partial moment model frees the investor from an asset pricing theory world where the general market index is the only appropriate component of the risky portion of an investor's portfolio. As the investor's financial situation changes over time, the LPM analysis can change with the investor. There are no compelling reasons to remain static with one utility function.

There is no empirical evidence during the past three decades supporting the concept of capital asset pricing theory (Fama and French, 1992). There is no evidence supporting market indexes as efficient investments (Haugen, 1990). There is no evidence that investor utility functions are irrelevant. When there is no supporting empirical evidence, there is no theory. Where there is no theory eliminating utility functions, we are left with the LPM(\mathbf{a}, \mathbf{t}) model and its rich set of utility functions. Nobel laureate Richard Feynman (1964) in his famous Cornell University lecture series talks about how scientific laws and theories are developed.

"First, we make a guess. Then we compute the consequences of the guess. Then we compare the consequences to experiment. If it disagrees with experiment, it's wrong. In that simple statement is the key to science. It doesn't make a difference how beautiful your guess is. It doesn't make a difference how smart you are, who made the guess, or what his name is. If it disagrees with experiment, it's wrong."

The last statement should be framed and placed on the wall of every research office in the country.

The previous discussion may need some clarification. Very simply, there are two issues. First, the economic utility of individual investors has to be brought back into the investment analysis after being eliminated by capital asset pricing theory. Second, having said that, economic utility does not have to be applied exactly or optimally. Roy(1952) solves the problem of maximizing utility by using an approximate measure, the safety first principle. Utility does not have to be maximized. Economic outcomes only have to satisfy the investor. Herb Simon won a Nobel Prize in economics in 1979 by telling us that utility only has to be “satisficed” not maximized (1955). The Lower Partial Moment measure based on the Roy safety first principle can be either an appropriate utility satisficing measure or a utility maximizing measure.

The LPM(**a,t**) does not have to be applied precisely, as it is flexible enough to be applied heuristically as a person would actually make a decision. The key question is whether the LPM measure by measuring financial asset distributions more accurately is providing a better forecast of the future than other alternatives such as the variance. I think it does.

Do academic approaches have value to practitioners? The academic approach helps the analyst understand the system with which he/she is interacting. Understanding the why, where, and how is superior to trying to follow a black box solution. The worst thing a practitioner can ask is, “Give me a rule of thumb that I can follow without thinking.” In most cases, the rule does not exist. In addition, most forecasting techniques derive from the academic approach to understanding the system. Once, the academic world has developed an understanding of the system, then practical heuristic decision techniques will follow. Roy developed the safety first principle as a heuristic because he wanted a practical application of utility theory. Practitioners and academics have to learn to understand each other. The academic is almost always looking backward seeking to understand and explain. The practitioner is looking forward seeking to forecast. It would be nice if the two would listen to each other and help each other to understand the complete picture both forward and backward.¹¹

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Notes

1. Bernstein (1993), p 41.
2. The ability to maximize the economic utility of an individual by maximizing one mathematical expression is still an issue today.
3. The d value in Sharpe's R/V ratio is the riskless rate of return. Because it is a Roy safety first R/V ratio, the Sharpe R/V ratio could be computed with any d value. Sharpe (1966) chose the Treasury bill return as an appropriate safety first return. Later Rom and Ferguson (1994a) use the loss threshold as their safety first ($d = 0$) return in computing the Sharpe R/V ratio and are admonished by Kaplan and Siegel (1994a, footnote 9) for not using the riskless rate of return. However, it is appropriate to compute the R/V ratio using their minimum acceptable return (MAR) just as they use the MAR in their R/SVt ratio. The MAR is the Roy Safety First return. Rom and Ferguson (1994b, footnote 19), however, are incorrect when they state that the results of using R/V ratio to find the portfolio with the best risk-return performance are independent of the MAR. As the MAR varies, different portfolios on the EV frontier will have the maximum R/V ratio.
4. Naming a statistic after a researcher is very tricky as it implies that the statistic has not been used previously. The R/SVt ratio is a reward to below-target semideviation version of the Roy Safety First R/V ratio. This ratio has been in use for three decades by academics as a variant of the Roy Safety First and as a result is in the public domain.
5. Balzer (1994) provides a very nice review of investment risk measures including a description of stochastic dominance.
6. The Philippatos E-S computer program remained unpublished until the U.S. Copyright law changed in 1978 to include the copyrighting of computer software. The E-S program is copyright © 1983 by George Philippatos and David Nawrocki. The E-S and E-V algorithms are implemented in the Portfolio Management Software Package (PMSP) that has been marketed by Computer Handholders, Inc. since 1982. PMSP is the first commercially available software package to provide an optimizer for below-target semivariance risk measures.
7. Contrary to Balzer (1994, p.57), there is no lower partial skewness ($a=3$) or lower partial kurtosis ($a=4$). Specifically, there are no negative values when the below-target returns are cubed so a partial skewness value is not possible. As Balzer notes, the a value is simply a risk aversion coefficient in these cases. As such it does not represent a traditional third or fourth moment of the distribution. In addition, there has been research by Fishburn (1977) on these higher order measures and their equivalence to stochastic dominance. The a -degree Lower Partial Moment in all of its forms (up to a degree $a = 15$ is reacting to the amount of skewness and dispersion in the distribution (Nawrocki, 1990,1992). The higher the degree, a , of the LPM, the higher the skewness preference in the LPM utility function.
8. Kaplan and Siegel (1994a) received strong counter arguments from Rom and Ferguson (1994b). They bury Kaplan and Siegel's arguments under a virtual avalanche of quotes from researchers in downside risk measures. To be fair, Rom and Ferguson's Exhibit 15 and Exhibit 16 do not properly represent Kaplan and Siegel's arguments. Exhibit 15 demonstrates a utility function that utilizes put positions (Rom and Ferguson, 1994b, footnote 16) not a characterization of Fishburn utility functions by Kaplan and Siegel. Exhibit 16 is splitting hairs since the quadratic utility function has never been represented anywhere in the literature by a true quadratic parabola curve. Overall, the literature supports Rom and Ferguson. Balzer (1994) provides an independent review of risk measures and concludes the evidence leads to the use of the semivariance. Merriken (1994) argues that there are many investors with short-term time horizons who would be best served by the semivariance measure. Since Fishburn (1977) demonstrates that $a = 1.0$ is risk neutral behavior, the LPM(a,t) model also adapts to long term strategies. As shown here, the LPM measure is strongly grounded in utility theory and is appropriate for individuals.
9. Kaplan and Siegel (1994a, footnote 7) state that skewness and kurtosis are independent of the observer and any resulting utility function. Tables 4 and 5 indicate that the investor can use a utility

function to control the amount of skewness in a portfolio. Therefore, there is a relationship between the observer, the observer's utility, and the skewness of the return distribution.

10. This enters into the argument as to whether human behavior can be captured in one mathematical utility function. Currently, it cannot be done.
11. If you have the feeling after reading this paper that academics thought of everything first, take the hint. Learn to read the academic literature. However, as seen with asset pricing theories, academics are not always right and we tend to be very dogmatic with our explanatory viewpoints. However, when faced with a question on how to approach a problem, there is a very good chance that an academic has already considered and developed an understanding of the problem. You don't need to reinvent the wheel.

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Short Biography of David Nawrocki

David N. Nawrocki is professor of finance at Villanova University. He holds an MBA degree and a Ph.D. degree in finance from the Pennsylvania State University. Dr. Nawrocki is the author of a portfolio optimization package (PMSP Professional) marketed since 1982 by Computer Handholders, Inc. and is the director of research for The QInsight Group, an investment management firm. His research articles have appeared in journals such as *Journal of Financial and Quantitative Analysis*, *The Financial Review*, *The International Review of Financial Analysis*, *Journal of Business Finance and Accounting*, *Applied Economics*, and *the Journal of Financial Planning*.

Summary of A Brief History of Downside Risk Measures

There has been a controversy in this journal about using downside risk measures in portfolio analysis. The downside risk measures supposedly are a major improvement over traditional portfolio theory. That is where the battle lines clashed when Rom and Ferguson (1993, 1994b) and Kaplan and Siegel (1994a, 1994b) engaged in a "tempest in a teapot." One of the best means to understand a concept is to study the history of its development. Understanding the issues facing researchers during the development of a concept results in better knowledge of the concept. The purpose of this paper is to provide an understanding of the measurement of downside risk through tracing its development from 1952 and the initial portfolio theory articles by Markowitz and Roy through to the Rom and Ferguson-Kaplan and Siegel controversy in 1994.